Evolutionary Game Theory

- EGT turns non-cooperative GT on its head
- Instead of assuming all players are fully rational, let us assume that they are automats. They are born (pre-programmed) to play a certain pure or mixed strategy, matched randomly with an opponents, and then reproduce and spread their programs (genes).
- We will first develop an intuition for the stability of such systems, by analyzing the dynamics of sample evolutionary system,
- We will then develop a test (refinement) of Nash equilibrium – an Evolutionary Stable Strategy

Replicator dynamics

- Stage 1: Animals are born behaviorally ‘pre-programmed’
- Stage 2: Each animal is matched with another in the cohort and interacts. The behavioral programs determine the payoff from interaction: reproductive capabilities of each animal
- Stage 3. Animals reproduce their children inherit the behavioral program
- The cycle repeats itself ad infinum.
- The total population is usually assumed constant *N* (enough food for *N* animals), but what really matters are the fractions of the population that carry the specific replicator

Payoffs

- Time is discrete, τ is the length of life cycle
- There are 2 behavioral programs, and the reproductive capabilities are a product of the payoff (see payoff matrix) and the length of the cycle

	Soft (S)	Aggressive (A)
Soft (S)	u+1, u+1	u+0, u+2
Aggressive (A)	u+2, u+0	u-1, u-1

- $p(t) = p$ = fraction of pop. with A-replicator at the beginning of period t
- $N_A(t+\tau)$ = # of animals with A-replicator at the end of t
- $p(t+\tau) = N_A(t+\tau)/ N(t+\tau)$ - fraction of pop. with A-replicator at the end of period t

Looking for stable states

$$N_A(t+\tau) = Np\{1+\tau[p(u-1)+(1-p)(u+2)]\} = Np\{1+\tau[-p+u+2-2p]\} =$$
$$= Np\{1+\tau[u+2-3p]\} = Np(1+\tau y)$$

$$N_S(t+\tau) = N(1-p)\{1+\tau[pu+(1-p)(u+1)]\} = N(1-p)\{1+\tau[u+1-p]\} = N(1-p)(1+\tau x)$$

$$p(t+\tau) = \frac{Np\{1+\tau y\}}{Np\{1+\tau y\}+N(1-p)\{1+\tau x\}}$$

We want to see how the fractions change with time, find the difference:

$$p(t+\tau)-p(t) = \frac{p(1+\tau y)-p[p(1+\tau y)+(1-p)(1+\tau x)]}{p(1+\tau y)+(1-p)(1+\tau x)}$$

Still looking

$$\frac{p(t+\tau)-p(t)}{\tau} = \frac{p(1+\tau y)-p[p(1+\tau y)+(1-p)(1+\tau x)]}{\tau[p(1+\tau y)+(1-p)(1+\tau x)]} =$$

$$= \frac{p+p\tau y-p^2-p^2\tau y-p-p\tau x+p^2+p^2\tau x}{\tau[p(1+\tau y)+(1-p)(1+\tau x)]} =$$

$$= \frac{p\tau y-p^2\tau y-p\tau x+p^2\tau x}{\tau[p(1+\tau y)+(1-p)(1+\tau x)]} = \frac{p(y-py-x+px)}{[p(1+\tau y)+(1-p)(1+\tau x)]}$$

- The limit of the above as τ goes to 0 is the derivative of p(t), it therefore describes the dynamics of the system:

$$p'(t) = \frac{p(y-py-x+px)}{p+(1-p)} = p(y-py-x+px) = p(1-p)(y-x) =$$

$$= p(1-p)(u+2-3p-u-1+p) = p(1-p)(1-2p)$$

Stability

- There are 3 roots of this equation: 0, 1, ½
- They correspond to NE of the stage game. Notice that if 1 > p >1/2 then p'(t) < 0, but if 1/2 > p > 0, then p'(t) > 0. (Graph p(t) vs. t)
- If for example, the whole population consists of A types, the system will stay at p(t) = 1 forever
- However, if there is a mutation, the mutant S-type animal will get much greater payoff than the A-types, and will replicate faster.
- Same about p(t)=0. The 'pure-strategy' sets are therefore steady but locally unstable.
- The mixed strategy equilibrium is globally stable: if there is any type of mutation, the system will approach p(t) = ½ in the long run.

Evolutionary Stable Strategies

- The replicator dynamics can be applied to any symmetric game
- A priori, we may not know what strategy the opponent will choose. But we can check which strategy is stable in the evolutionary sense
- Notice that only symmetric NE can be stable
- Take a symmetric NE. Notice that a mutation that is not a best response to the equilibrium strategy will die out.
- A **strategy** b^* **is evolutionary stable** if (b^*, b^*) is a Nash equilibrium of the (symmetric) game and the expected payoff $u(b, b) < u(b^*, b)$ for any strategy $b \neq b^*$ that is a best response to b^*

ESS

- In the above game, only a mixed strategy, where soft is played with prob. ½ is evolutionary stable.
- In the Hawk-Dove game below, the strategy ‘Hawk’ is ESS.

	Dove	Hawk
Dove	½ , ½	0, 1
Hawk	1, 0	¼ , ¼

ESS

- In the game below, the NE is a mixed strategy (1/3, 1/3, 1/3), but there is no ESS

		Player 2			
		L	M	R	
	L	½,½	1, -1	-1, 1	
Player 1	M	-1, 1	½,½	1, -1	
	R	1, -1	-1, 1	½,½	

REVIEW

- Choice Under Uncertainty:
  - check if choices are consistent with the independence assumption
  - check if a lottery is 1st or 2nd order dominated by another
- Normal Form Games:
  - find solution by IEDS (including auctions)
  - find NE (including mixed-strategy)
  - state theorems for games with incomplete information
- Extensive Form Games:
  - construct a game tree, given the description of a game
  - translate a game tree into a normal form
  - find a SPNE
  - prove (disprove) that a pair of strategies is SPNE in a supergame
- Cooperative Game Theory:
  - find the Core of a coalitonal game
  - find SV of a coalitional game